Abstract: Over the last three decades a vast literature has been dedicated to supervenience. Much of it has focused on the analysis of different concepts of supervenience and their philosophical consequences. This paper has two objectives. One is to provide a short, up-to-date, guide to the formal relations between the different concepts of supervenience. The other is to reassess the extent to which these concepts can establish metaphysical theses, especially about dependence. The conclusion is that strong global supervenience is the most advantageous notion of supervenience that we have.

1. Introduction

In "Concepts of Supervenience" Jaegwon Kim (1984) characterizes and distinguishes among three concepts of supervenience: strong, weak and global. The initial focus was the "local" notions of strong and weak supervenience. Gradually however, the notion of global supervenience has taken center stage. It has been noticed that there are different versions of global supervenience, and that these versions have interesting philosophical consequences. It has been argued, for example, that strong global supervenience supports psycho-physical dependencies in the context of externalist taxonomies of mental states.\(^1\) It has also been argued that the weaker versions of global supervenience can be used to support various metaphysical theses, for example, the thesis that there exist coincident but numerically distinct entities.\(^2\)

The paper has two objectives. One is to provide a short guide to the relations between concepts of supervenience (Section 2). The guide focuses on the varieties of global supervenience; definitions and results are mostly taken from Kim (1984, 1987, 1990), Paull and Sider (1992), Stalnaker (1996), McLaughlin (1997), Sider (1999),

\(^1\) See Shagrir (2002); for earlier versions of this claim that refer to global supervenience, see Kim (1987) and Petrie (1987).
Shagrir (2002) and Bennett (2004). The other objective is to reassess the extent to which the different notions of supervenience can establish metaphysical theses, especially about dependence. In section 3 I review and defend the claim that the weaker notions of global supervenience do not establish dependence. In section 4 I discuss and rebut three recently advanced arguments against the strong version of global supervenience (Hoffmann and Newen 2007; Leuenberger 2009; Bennett 2004). My conclusion is that strong global supervenience is the most viable notion of supervenience we have.

2. Varieties of supervenience: A tour guide

Supervenience is a relation between sets A and B of properties and relations. Global supervenience was first introduced by Horgan (1982) as a relation between the B-properties and A-properties of all the objects in any possible world.\(^3\) It states that any two worlds that are B-indiscernible are also A-indiscernible. It turns out, however, that we can understand this indiscernibility condition in different ways. Stalnaker (1996) and McLaughlin (1997) sharply distinguish between strong and weak flavors of global supervenience. Bennett (2004) and Shagrir (2002) introduce the notion of intermediate supervenience, arguing that it is the notion of global supervenience that most philosophers had in mind. I follow here Sider's (1999) definition style, which takes into account not only properties but also relations.

We start with two more general definitions:

\(^3\) Horgan also introduces the notion of regional supervenience, and distinguishes between strong and weak versions of this notion (see also Horgan 1993: 570-572).
• Two possible worlds, \( w_i \) and \( w_j \) are \( X \)-indiscernible \( \equiv \) \( \text{def.} \). There is an \( X \)-isomorphism from the domain of (i.e., set of objects existing at) \( w_i \) onto the domain of \( w_j \).

• A function \( f \) is an \( X \)-isomorphism \( \equiv \) \( \text{def.} \) if \( f \) is one-to-one, and for any \( n \)-place relation \( R \) in \( w_i \) and for any \( n \) objects in \( f \)'s domain, \( w_i, R(a_1, \ldots a_n) \iff R(f(a_1), \ldots f(a_n)) \).

We can now define the three versions of global supervenience:

• **Weak global supervenience** (WGS): A weakly globally supervenes on \( B \) \( \equiv \) \( \text{def.} \). Any two possible worlds that are \( B \)-indiscernible are also \( A \)-indiscernible.

• **Intermediate global supervenience** (IGS): A intermediately globally supervenes on \( B \) \( \equiv \) \( \text{def.} \). For any two \( B \)-indiscernible worlds, \( w_i \) and \( w_j \), there is some \( B \)-isomorphism \( f \) from the domain of \( w_i \) onto the domain of \( w_j \) that is also an \( A \)-isomorphism.

• **Strong global supervenience** (SGS): A strongly globally supervenes on \( B \) \( \equiv \) \( \text{def.} \). For any two worlds, \( w_1 \) and \( w_2 \), any \( B \)-isomorphism from \( w_i \)'s domain onto \( w_j \)'s domain is also an \( A \)-isomorphism.

It is obvious that \( \text{WGS} \leq \text{IGS} \leq \text{SGS} \), namely, that SGS entails IGS, which, in turn, entails WGS. But it can be shown that \( \text{WGS} < \text{IGS} < \text{SGS} \), namely, that WGS does not entail IGS, and IGS does not entail SGS. Let us first see that \( \text{WGS} < \text{IGS} \).

**Theorem 1** (Sider 1999): Scenario (I) is consistent with the WGS of \( \{Q\} \) on \( \{P\} \).

(I) \[ \begin{align*}
\quad w_1 & \quad w_2 \\
\quad Pa & \quad Qa & \quad Pc & \quad \neg Qc \\
\quad \neg Pb & \quad \neg Qb & \quad \neg Pd & \quad Qd
\end{align*} \]

The idea here is that \( w_1 \) and \( w_2 \) are \( \{P\} \)-indiscernible: there is a \( \{P\} \)-isomorphism from \( w_1 \) onto \( w_2 \) that maps a onto c and b onto d. But \( w_1 \) and \( w_2 \) are also \( \{Q\} \)-indiscernible: the function that maps a onto d and b onto c is a \( \{Q\} \)-isomorphism from \( w_1 \) onto \( w_2 \). Paull and Sider (1992) taught us, however, that this explanation is not sufficient. Global supervenience is defined over all possible worlds, and we have no
guarantee that in this wider context WGS allows such worlds as \( w_1 \) and \( w_2 \). So here is a more general explanation:

Let us define \( Q \) as follows: \( Qx = \exists y Py \land \exists z (x \neq z \land \neg Qz) \). \(^4\) \( \{Q\} \) weakly globally supervenes on \( \{P\} \). It follows from the definition that \( Q \) is instantiated only in worlds with at least one \( P \)-object (object that has \( P \)) and exactly one \( \neg Q \)-object. In any such world, all the other objects, if there are any, are \( Q \)-objects. Now, take any two \( \{P\} \)-indiscernible worlds, \( w_i \) and \( w_j \). It follows from the \( \{P\} \)-isomorphism that \( w_i \) and \( w_j \) have the same number of objects and the same number of \( P \)-objects. If \( w_i \) (and so \( w_j \)) include no \( P \)-objects, or fewer than two objects, then there are no \( Q \)-objects in \( w_i \) and in \( w_j \), and \( w_i \) and \( w_j \) are \( \{Q\} \)-indiscernible. In all other cases, one of the objects in \( w_i \) (and so in \( w_j \)) is a \( \neg Q \)-object, and the others, if there are any, are \( Q \)-objects. Thus a one-to-one function from \( w_i \) onto \( w_j \) that maps the \( \neg Q \)-object of \( w_i \) onto the \( \neg Q \)-object of \( w_j \) is a \( \{Q\} \)-isomorphism. Thus \( w_i \) and \( w_j \) are \( \{Q\} \)-indiscernible.

**Theorem 2**: Scenario (I) is inconsistent with the IGS of \( \{Q\} \) on \( \{P\} \).

Worlds \( w_1 \) and \( w_2 \) are \( \{P\} \)-indiscernible. Yet there is no \( \{P\} \)-isomorphism that is also a \( \{Q\} \)-isomorphism. The only \( \{P\} \)-isomorphism from \( w_1 \) onto \( w_2 \) is that which maps \( a \) onto \( c \) and \( b \) onto \( d \). But this \( \{P\} \)-isomorphism is not a \( \{Q\} \)-isomorphism.

**Corollary**: WGS<IGS.

Let us now turn to see that IGS<SGS.

**Theorem 3** (Bennett 2004; Shagrir 2002): Scenario (II) is consistent with the IGS of \( \{Q\} \) on \( \{P\} \).

(II) \[
\begin{array}{ll}
w_1 & \quad w_2 \\
Pa & \& Qa & \quad Pc & \& Qc \\
Pb & \& \neg Qb & \quad Pd & \& \neg Qd \\
\end{array}
\]

The idea is that the function that maps \( a \) to \( c \) and \( b \) to \( d \) is both a \( \{P\} \)-isomorphism and a \( \{Q\} \)-isomorphism from \( w_1 \) onto \( w_2 \). A more general explanation is this:

\(^4\) Alternatively, one can think of the equation as a *description* of the relations between \( P \) and \( Q \) in all possible worlds.
Let us define the property Q as follows: Qx =df Px & \exists y (y ≠ x & Py & ¬Qy). \{Q\}
intermediately globally supervenes on \{P\}: It follows from the definition of Q that (i) all Q-objects are also P-objects; (ii) that in a world with exactly one P-object, this object has ¬Q; and (iii) that in any worlds with at least two P-objects, one P-object has ¬Q, and the others have Q. Now, take any two \{P\}-indiscernible worlds, w_i and w_j. Let n be the number of P-objects in each of the two worlds. If n≥2, then in each of the two worlds, one P-object has ¬Q, the other P-objects have Q, and all the ¬P-objects have ¬Q. Thus any one-to-one function from w_i onto w_j that maps the P-object which has ¬Q in w_i onto the P-object that has ¬Q in w_j, and any of the other n-1 P-objects in w_i onto any of the other n-1 P-objects in w_j, is both a \{P\}-isomorphism and a \{Q\}-isomorphism from w_i onto w_j. If n<2, then all objects in both worlds have ¬Q. Thus the \{P\}-isomorphism from w_i onto w_j is, trivially, a \{Q\}-isomorphism.

**Theorem 4** (Bennett 2004; Shagrir 2002): Scenario (II) is inconsistent with the SGS of \{Q\} on \{P\}.

There is a \{P\}-isomorphism from w_1 onto w_2 that is not a \{Q\}-isomorphism. The function from w_1 onto w_2 that maps a onto d and b onto c is a \{P\}-isomorphism, but not a \{Q\}-isomorphism.

**Corollary** (Bennett 2004; Shagrir 2002): IGS<SGS.

Let us turn to the relations between the global notions of supervenience and the two individual ("local") notions, of weak and strong supervenience. I start with definitions:

- **Strong individual supervenience (SIS):** A strongly individually supervenes on B =_def. Any two possible objects that are B-indiscernible (i.e., have exactly the same B-properties) are also A-indiscernible.\(^5\)

- **Weak individual supervenience (WIS):** A weakly individually supervenes on B =_def. Any two B-indiscernible objects of the same world (any world) are also A-indiscernible.\(^6\)

\(^5\) Here is Kim's (1987) version: A-properties strongly supervene on B-properties if and only if for any possible worlds w_1 and w_2 and any objects x in w_1 and y in w_2, if x in w_1 is B-indiscernible from y in w_2, then x in w_1 is A-indiscernible from y in w_2.

\(^6\) Here is Kim's (1987) version: A-properties weakly supervene on B-properties if and only if for any possible world w and any objects x and y in w, if x and y are B-indiscernible in w, then they are A-indiscernible in w.
It should be noted that SIS and WIS are defined over intrinsic properties of x, such as Px, and extrinsic properties, such as ∃y(y≠x & Rxy), but not relations.

We thus restrict our comparisons to intrinsic and extrinsic properties alone, and will return to relations at the very end of the paper.

**Definition:** \( S1 \leq_p S2 \) just in case \( S2 \) entails \( S1 \) when the sets A and B are restricted to properties.

**Definition:** \( S1 <_p S2 \) just in case \( S1 \leq_p S2 \), but \( S1 \) does not entail \( S2 \) when the sets A and B are restricted to properties.

**Theorem 5:** WIS does not entail IGS (IGS not-\( \leq_p \) WIS).

The WIS of \( \{Q\} \) on \( \{P\} \) is consistent with scenario (I), but not with the IGS of \( \{Q\} \) on \( \{P\} \) (theorem 1).

**Theorem 6:** IGS does not entail WIS (WIS not-\( \leq_p \) IGS).

The IGS of \( \{Q\} \) on \( \{P\} \) is consistent with scenario (II) (theorem 3), but not with the WIS of \( \{Q\} \) on \( \{P\} \): the objects a and b of \( w_1 \) have the same \( \{P\} \)-properties but different \( \{Q\} \)-properties.

**Theorem 7:** WIS \( \leq_p \) SGS.

Assume the SGS of A on B (A and B are sets of properties), and take two B-indiscernible objects, a and b, in some world, w. We should show that the objects are also A-indiscernible. The function f from w onto w that maps a onto b, b onto a, and every other object onto itself is a B-isomorphism. From SGS it follows that f is also an A-isomorphism. Thus a and b are A-indiscernible.

**Theorem 8:** WIS \( <_p \) SGS (WIS does not entail SGS).

WIS \( \leq_p \) SGS (theorem 7). Scenario (I) is consistent with the WIS of \( \{Q\} \) on \( \{P\} \), but is inconsistent with the SGS of \( \{Q\} \) on \( \{P\} \).

**Theorem 9:** SGS \( \leq_p \) SIS.

Assume that f is a B-isomorphism from \( w_1 \)'s domain onto \( w_2 \)'s domain. Thus any object x in \( w_1 \) has a counterpart x' in \( w_2 \) with exactly the same B-properties. From the strong individual supervenience of A on B, it follows that x and x' must also have the same A-properties. Thus f is also an A-isomorphism.
Theorem 10: SGS \preceq_p SIS (SGS does not entail SIS), when B is restricted to intrinsic properties.

SGS \preceq_p SIS (theorem 9), and Paull and Sider (1992) prove that the following scenario (III) is consistent with the SGS of \{Q\} on B, but inconsistent with the SIS of \{Q\} on B. Here B = \{P,S\} and Qx = Px & \exists ySy, and the scenario is the following:

(III) \ w_1 \ w_2
    Pa & \neg Sa & Qa
    & Sc & \neg Pc & \neg Qc
    Pb & \neg Sb & \neg Qb

\{Q\} does not strongly individually supervene on B, because a and b have the same B-properties (they both have P and do not have S) but different \{Q\}-properties (Paull and Sider 1992: 840-841). The scenario, however, does not rule out the SGS of \{Q\} on B, for there is no B-isomorphism from w_1 onto w_2. Paull and Sider (1992: 852-853) prove that \{Q\} strongly globally supervenes on B.

It turns out that SGS and SIS are equivalent in several important cases. Bennett (2004: 524-525) proves that SGS and SIS are equivalent when the set A is restricted to intrinsic properties and the set B is restricted to intrinsic and extrinsic properties. Stalnaker (1996) proves that the strong global supervenience of A on B is equivalent to the strong individual supervenience of A on the set of maximal B-properties. To prove Stalnaker's theorem, let us first characterize maximal properties.

Definition (Stalnaker 1996; Sider 1999): The complete world perspective of an object x in terms of B-properties is known as x’s maximal B-property. It can be described by an open formula that mentions every property of x and its worldmates, and every relation in x’s world (for further details see Stalnaker 1996: 238, and Sider 1999: 919-920). For example, this would be the formula that expresses the maximal \{P\}-property of the object a in scenario (I): x has property P; there is only one object, y, in the world other than x; y does not have P.
Theorem 11 (Stalnaker 1996; see also Bennett 2004 and Shagrir 2002): The strong global supervenience of A on B is equivalent to the strong individual supervenience of A on the set of maximal B-properties (the sets A and B are restricted to properties).

Assume the SGS of A on B. Take any two possible objects x and x' with the same maximal B-properties. Since a maximal B-property provides a complete description of the B-properties in a world, there must be a B-isomorphism, $f$, from the domain of x’s world onto the domain of x’’s world, and $f(x)=x'$. It follows from SGS that $f$ is also an A-isomorphism. Thus, x and x' have the same A-properties.

For the other direction: assume the SIS of A on the set of maximal B-properties. Take any two B-indiscernible worlds, $w_i$ and $w_j$, and any B-isomorphism $f$ from $w_i$ onto $w_j$. We should show that $f$ is also an A-isomorphism from $w_i$ onto $w_j$.

Take any A-property, $A'$, and any object x with $A'$. It follows from the B-isomorphism that x and $f(x)$ have exactly the same maximal B-properties. Also, it follows from SIS that x and $f(x)$ have the same A-properties. It thus follows that $f(x)$ has $A'$. Hence $f$ is also an A-isomorphism.

3. Supervenience and dependence: WIS, WGS and IGS

In this section I review the current status of the debate on whether the weaker types of supervenience (WIS, WGS, and IGS) establish dependence. The conclusion is that they do not, at least in the context of the way we think about objects and properties. Indeed, there is good reason to claim that WIS, WGS and IGS cannot secure dependence. Both WIS and WGS are consistent with scenario (I), namely, with a scenario in which I and my twin (who inhabits another world) are physically alike in all our intrinsic and extrinsic properties, but radically differ in our mental properties. Both WGS and IGS are consistent with scenario (II), namely, with a scenario in which I and my twin (who inhabits the same world) are physically alike in all our intrinsic and extrinsic properties, but radically differ in our mental properties. Doesn't this, by itself, show that we cannot count on WIS, WGS and IGS to establish dependence?
The answer is not straightforward. First, there are contexts in which the weaker notions of supervenience are strong. Moyer (2008) argues that WIS is equivalent to SIS, when the set A is restricted to intrinsic properties. Bennett (2004: 524-525) proves that WGS and IGS are equivalent to SGS (and SIS) when the set A is restricted to intrinsic properties.\(^7\) Second, Sider (1999) notes that WGS is committed to cross-world relations, for example, it implies that in every world that is \(\{P\}\)-indistinguishable from \(w_1\) (in (I)), one individual has Q, and the other \(\neg Q\). The stronger IGS is also committed to such cross-world relations; WIS, however, is not committed to this.

To demonstrate the philosophical force of WGS in establishing dependence, Sider invokes the debate over coincident entities. Coincidentalists claim that two entities, say a statue and a lump of matter, are numerically distinct, even if made out of the same parts. Thus Goliath and the lump of matter (Lumpl) share the same non-modal properties and relations. They differ only in their modal properties: Lumpl, but not Goliath, might have survived being squashed.\(^8\) It has been argued against coincidentalism that the modal properties of Goliath and Lumpl must be "grounded" in non-modal properties. But if "grounded" means, or implies, supervenience, then Lumpl and Goliath, which are non-modally indiscernible, cannot differ in their modal properties.

Sider (1999) replies to this argument on behalf of coincidentalism. He points out that if supervenience is of the weak global variety, then the argument fails. As scenario (II) indicates, the WGS (and the IGS) of the modal on the non-modal is

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\(^7\) Bennett assumes an "isolation principle" introduced in Paull and Sider (1992); I also appeal to this principle below. Moyer (2008) assumes a recombination principle that, roughly, adds on to the isolation principle the possibility of making up worlds from isolated parts of other worlds.

\(^8\) See Gibbard (1975).
compatible with Lumpl and Goliath having the same non-modal properties and relations, but different modal properties. WGS is committed only to the claim that in another world, which is non-modally indistinguishable from ours, the two objects that are non-modally indistinguishable from Lumpl and Goliath differ in their modal properties: one might have survived being squashed, the other not. But this commitment can please the coincidentalist.

Bennett (2004) and I (Shagrir 2002) respond to Sider, arguing that WGS and IGS cannot establish dependence. Scenario (II) really indicates that the modal weakly globally supervenes on the non-modal, though the modal does not depend on the non-modal. For in this scenario, a and b have exactly the same maximal \{P\}-properties, but different \{Q\}-properties. Thus nothing in their \{P\}-properties, intrinsic or extrinsic, determines or explains why one has Q and the other not, why it is the object a and not the object b that has Q. And this shows that at least something about the \{Q\}-properties is not dependent on the \{P\}-properties. The same goes for Goliath and Lumpl, which have exactly the same maximal non-modal properties, but different modal properties. Nothing in their non-modal properties, whether intrinsic or extrinsic, determines or explains their modal difference, namely, why it is Lumpl and not Goliath that might have survived being squashed.

The argument assumes a more general principle, according to which a minimal requirement on dependence of A on B is *entailment* from the maximal B-properties of an object to its A-properties. Entailment means that the B*x → Ax conditionals will be true necessarily, that is, for any possible x (B* is the maximal B-property of x). WIS, WGS, and IGS do not satisfy this requirement, as indicated by scenarios (I) and (II). They are all compatible with scenarios in which two possible objects have
exactly the same B-properties but different A-properties. But is this entailment principle justified? One could say that WGS does satisfy a weaker entailment condition on which the instantiation of B* entails the instantiation of A by some object, even if not by the object that has B*. Coincidentalists can live with this condition, as they also agree that the instantiation of non-modal properties entails the instantiation of modal properties, i.e., that one object in the world might have survived being squashed, and another not. As one might note, I have not given reasons for accepting the stronger entailment requirement.⁹

The aim of the argument, however, is not so much to give reasons for entailment as to highlight the metaphysical assumptions that underlie our notion of dependence. The crucial metaphysical assumption is that properties are instantiated by objects: monadic properties are instantiated by singleton objects, relations by pairs of objects, etc. We thus expect that if A depends on B, then there is something about B which determines that, or explains why, this very object instantiates (or not) an A-property. In particular, we take the claim that the modal depends on the non-modal to mean that non-modal properties determine that, or explain why, this very object is Goliath and the other one is Lumpl, and not the other way around. This does not mean that whether something is Goliath and not Lumpl is determined by its non-modal properties. It might well be that the modal properties of the object are determined by none of its ("intrinsic") non-modal properties. The point, rather, is that the modal properties of an object should be determined by some non-modal properties (the use of maximal B-properties is a matter of convenience, as it provides a complete world description of the distribution of B-properties, by this object and all others). And the

⁹ This objection was raised by Mark Moyer (private communication).
problem here is that no non-modal property can distinguish between Goliath and Lumpl. The coincidentalist can certainly challenge these assumptions. But it takes more than merely indicating that WGS is committed to cross-world relations between A-properties and B-properties.\footnote{More recently, Sider (2008) concedes that the supervenience of the modal on the non-modal should be of the \textit{strong} global variety, and he advances a different reply on behalf of coincidentalism. Roughly, he suggests that there is a \textit{basic modal relation} of Goliath and Lumpl of \textit{opposite-possibly surviving being squashed}, meaning that exactly one might have survived being squashed.}

4. \textbf{In defense of SGS}

Unlike the weaker notions, SIS and SGS satisfy the requirement of entailment. Kim (1984) proves that entailment is satisfied by SIS, and Sider (1999) proves that it is satisfied by SGS.\footnote{But see Donato and Polanski and (2006), who challenge some premises of the proofs.} But there are other concerns about SGS. I address and defend SGS against three such concerns: (a) Hoffmann and Newen's (2007) claim that SGS cannot distinguish between relevant and irrelevant properties, and that, hence, it is too weak to establish dependence (even though satisfying entailment); (b) Leuenberger's (2009) argument that SGS does not capture an essential feature of global supervenience; and (c) Bennett's (2004) contention that "every claim made with SGS can also be made with strong supervenience [SIS]", and, hence, that "SGS has no metaphysically distinctive role to play" (p. 506).\footnote{Another concern stems from Kim's argument that no notion of supervenience establishes dependence, since supervenience "merely states a pattern of property covariation" (1998:10)", and "property covariation \textit{per se} is metaphysically neutral" (Kim 1990:148). Thus supervenience "leaves open the question of what \textit{grounds} or \textit{accounts for} it--that is, why the supervenience relation obtains" (1998: 9). Kim's point is well taken: by "establish" we should not mean that supervenience is a dependence relation, only that it reflects a dependence relation; see Shagrir (2009a).}
4.1 Hoffmann and Newen. Hoffmann and Newen (2007) are concerned with a notion of supervenience that does justice to externalism, the view that the individuation of mental types makes an essential reference to features in the individual's environment.\(^{13}\) They reason from scenarios such as (III) that SIS, when defined over intrinsic physical properties of objects, is not fit for the task. In this scenario, we recall, the extrinsic Q depends on \{P,S\}; however, SIS fails since the objects a and c have the same intrinsic properties, P, but one has Q and the other does not.

The problem with scenario (III) is solved if we switch to SGS, as \{Q\} strongly globally supervenes on \{P,S\}. Alternatively, we can *enlarge* the SIS base to include not only intrinsic properties, but also extrinsic properties of objects; indeed, following Stalnaker's theorem, we know that \{Q\} strongly (individually) supervenes on the maximal \{P,S\}-properties. Hoffmann and Newen worry, however, that SGS is *too weak*.\(^{14}\) Their reasoning is that the base now includes *all* intrinsic and extrinsic B-properties, including, in some cases, many irrelevant ones.\(^{15}\) SGS, they claim, does not distinguish between the relevant and irrelevant B-properties; according to it, all the B-properties are relevant:

The outlined problem results from the fact that according to strong global supervenience, higher-level properties depend on all features of the possible worlds under consideration, regardless of whether these are irrelevant to the instantiation of the higher-level property at issue. We hence label this difficulty the *irrelevant feature problem* (p. 311).

To demonstrate that SGS suffers from the *irrelevant feature problem*,

Hoffmann and Newen (pp. 308-309) describe the following scenario. Two worlds, \(w_1\)

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\(^{13}\) The classic papers that defend externalism are Putnam (1975) and Burge (1979).

\(^{14}\) Hoffmann and Newen raise a similar claim concerning SIS and maximal B-properties; I focus on their argument against SGS, but similar considerations apply to SIS.

\(^{15}\) By taking into account more B-properties you *strengthen* the requirement on B-isomorphism. But since this requirement is the antecedent of the supervenience conditional, it *weakens* the criterion on supervenience: e.g., supervenience is satisfied (vacuously) whenever there is *no* B-isomorphism.
and \( w_2 \), are physically the same, with one difference: the skyscrapers in John's world, \( w_1 \), are 2m higher, on average, than those in Paul's world, \( w_2 \). John and Paul are "physically alike" but have different mental states; John is tired and Paul is not (I assume that by "physically alike", Hoffmann and Newen mean that they are alike in intrinsic physical properties, as some extrinsic properties of John and Paul are surely different). However, the difference between them cannot be attributed to this irrelevant feature of the height of skyscrapers. Hoffmann and Newen conclude that they found a case on which (1) SGS holds, though (2) the mental does not (wholly) depend on the physical.

I first note that it would be a mistake to conclude from scenario (III) that either SIS (when defined over intrinsic B-properties) is too strong, or that SGS is too weak. A notion of supervenience is too strong/weak just in case there is a gap between the supervenience relations between two sets of properties and the dependence relations between the same sets of properties. In scenario (III), however, there are no such gaps. Both supervenience relations reflect the dependence relations between the properties over which they are defined: That \( \{Q\} \) does not strongly individually supervene on \( \{P,S\} \) reflects that \( \{Q\} \) does not depend on the intrinsic \( \{P,S\} \)-properties of individuals; and, that \( \{Q\} \) strongly globally supervenes on \( \{P,S\} \) reflects that \( \{Q\} \) does depend on the intrinsic and extrinsic (/maximal) \( \{P,S\} \)-properties of individuals. So there is no gap here between strong/individual supervenience and dependence.

Turning to Hoffmann and Newen's example, one could wonder why being tired is not dependent on the height of the skyscrapers; I'm more tired in the vicinity of skyscrapers. Hoffmann and Newen say that there is no dependence here, but saying is not enough. There are constraints on the notion of supervenience that must be met. In
particular, looking at \( w_1 \) and \( w_2 \) alone is insufficient to establish the strong global supervenience of the mental on the physical. SGS is being defined over all possible worlds, not just over \( w_1 \) and \( w_2 \). Thus attribution of properties in one world has consequences on their attributions in other worlds.\(^{16}\) Thus, Hoffmann and Newen should have looked at other worlds as well. Had they done that, they would have noticed that if the mental does not depend on the physical, then SGS does not hold.

Let us grant Hoffmann and Newen that being tired is not dependent on the height of skyscrapers. In this case, removing skyscrapers does not affect this mental state. According to Hoffmann and Newen, removing the skyscrapers also does not affect Paul's and John's intrinsic physical states (which are the same). This means (very roughly) that there is a possible world, \( w_3 \), that is physically like \( w_1 \), but the skyscrapers and their effects have been removed. In this world, John has a doppelganger, George, who is "physically alike" (same intrinsic physical properties), who is also tired. Similarly, there is a possible world, \( w_4 \), which is physically like \( w_2 \), but the skyscrapers and their effects have been removed. In this world, Paul has a doppelganger, Ringo, who, like Paul, is not tired. Now, \( w_3 \) and \( w_4 \) are physically indiscernible, for \( w_1 \) and \( w_2 \) are alike in all physical respects, excluding the skyscrapers. However, \( w_3 \) and \( w_4 \) are mentally discernible, since George is tired and Ringo is not. Thus, contrary to first appearance, SGS does not hold, in contrast to what Hoffmann and Newen assume.

To make the reply more precise, let us dramatize Hoffmann and Newen's example. Let us assume that the only physical difference between \( w_1 \) and \( w_2 \) is the height of one remote skyscraper in each world, which is very far away from John (and

\[^{16}\text{This point was made by Paull and Sider (1992) in their reply to Petrie (1987); see also Bennett and McLaughlin (2005).}\]
Paul). Still John is tired and Paul is not. Now let \( w_3 \) be \( w_1 \) minus the skyscraper. And let \( w_4 \) be \( w_2 \) minus the skyscraper. On Hoffmann and Newen's assumptions, \( w_3 \) and \( w_4 \) are physically indiscernible, and, in particular, George in \( w_3 \) and Ringo in \( w_4 \) are physically alike in their intrinsic and extrinsic properties. Here is the dilemma for Hoffmann and Newen: Do George and Ringo have the same mental states?

If the answer is "NO", then SGS does not hold, contrary to Hoffmann and Newen's assumption (1), and their example fails to challenge SGS. If the answer is "YES", then, I maintain, Paul's and John's mental difference is due to differences in the heights of the remote skyscrapers. Here is why: If the answer is "YES", then either George and John, or Ringo and Paul, have different mental states; let us assume that John is tired and George is not. What could account for the mental difference between John and George? We note that the only difference between the worlds, other than John's and George's mental states is the remote skyscraper. Moreover, if SGS holds, as Hoffmann and Newen assume, then it follows that in all possible worlds that are physically indistinguishable from \( w_1 \), John's doppelgangers are tired. SGS also implies that in all possible worlds that are physically indistinguishable from \( w_3 \), George's doppelgangers are not tired. The only physical difference between the two sets of worlds is the remote skyscraper, and in all these worlds this physical difference is correlated with exactly the same mental difference, of one individual being tired. Once you remove the skyscraper in all the worlds that are physically indistinguishable from \( w_1 \), the mental state of one person changes from being tired to not being tired.

All this indicates that, after all, being tired is dependent on the height of the remote skyscraper.

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17 This adjustment makes Hoffmann and Newen's story similar to Kim's wayward atom story (1989: 41), which meant to show that global supervenience is too weak to establish dependence. My reply to Hoffmann and Newen is along the lines of Paul and Sider's reply to Kim (pp. 841-847).
18 I adopt here Paul and Sider's isolation principle and other ideas of mereology.
skyscraper. We cannot really have stronger evidence for dependence than that. But then Hoffmann and Newen's assumption (2) is false.

The upshot is that in Hoffmann and Newen's example either the mental does not strongly globally supervene on the physical, or being tired is dependent on the height of skyscrapers. Their example having failed, Hoffmann and Newen are left with no support for their claim that SGS faces the irrelevant feature problem. My reply actually indicates that SGS does distinguish, at least in some cases, between relevant and irrelevant features; all we have to do is look at additional possible worlds.

4.2 Leuenberger. Stephan Leuenberger (2009) argues that SGS does not explicate the philosophical concept of global supervenience. According to Leuenberger, "it is a further principle of adequacy for an explication of global supervenience that it vindicates the following principle:

FPP If no two possible worlds are $B$-isomorphic, then every class of properties globally supervenes on $B$" (p. 120).\(^{19}\)

SGS, however, does not seem to satisfy the principle. Assume that the set $B$ includes fundamental properties, one of which is $P$, and also assume that $A$ is the class $\{Q\}$. Further assume that no two worlds share the same fundamental properties. Consider the following scenario:

\[(IV) \quad \begin{align*}
\text{w}_1 & \\
\text{Pa} & \land \text{Qa} \\
\text{Pb} & \land \neg \text{Qb}
\end{align*}\]

\(^{19}\)FPP stands for “Finest Partition Principle”.

According to FPP, A does globally supervene on B (assuming that no two worlds are B-isomorphic). But as scenario (IV) indicates A does not *strongly* globally supervene on B: the function that maps \(a\) to \(b\) and \(b\) to \(a\) is a B-isomorphism but not an A-isomorphism. Leuenberger thus concludes that SGS is *too strong* a notion for explicating global supervenience.

We can note first that the claim that FPP is a principle of adequacy for an explication of global supervenience conflicts with the claim that global supervenience is a notion of dependence. For as scenario (IV) shows, any notion of supervenience that satisfies FPP is not a notion of dependence. The objects \(a\) and \(b\) have the same fundamental B-properties but different A-properties; hence, their difference in A-properties is not grounded in the fundamental B-properties. The upshot is that it is a *good* thing that SGS does not satisfy FPP. After all, we want SGS to be a notion of dependence; yet it could not have been had it satisfied FPP. On the other hand, however, Leuenberger argues that it is a *bad* thing that SGS does not satisfy FPP: for satisfying FPP is a requirement of an adequate explication of global supervenience.\(^{20}\)

My aim in what follows is to challenge Leuenberger's argument.

Why think that satisfying FPP is necessary for the explication of global supervenience? One answer stems from the general consideration according to which something is true as long as it has not been refuted. In our context, we want to hold on to the global supervenience of A on B as long as we do not find a B-isomorphism between worlds that is not an A-isomorphism. In other words, it is a constraint on global supervenience that if no two worlds are B-isomorphic then A globally supervenes on B.

\(^{20}\) Leuenberger indeed concluded that there is tension in the philosophical concept of global supervenience.
I do not dispute this constraint. What is disputable is that FPP adequately expresses it. The friends of SGS can reply that FPP mentions only B-isomorphisms between different worlds, and not B-isomorphisms from a world onto itself. However, they might insist, B-isomorphisms from a world onto itself (other than identity mappings) are also relevant; these B-isomorphisms indicate something about the dependence relations between the distribution of the A-properties in a world and the distribution of the B-properties in this world (as scenario (IV) shows). The friends of SGS can thus conclude that we should extend FPP to include such cases:

**FPP** If there is no B-isomorphism from one world onto another world (including from a world onto itself, other than identity), then every class of properties globally supervenes on B

SGS, of course, does satisfy FPP*, and thus is in accord with global supervenience.

Another answer – the one provided by Leuenberger – is that global supervenience should allow "intraworld variation" of individuals: two objects that have the same B-properties but different A-properties. Leuenberger cites Bennett who writes that “everyone has always taken global supervenience to allow intraworld variation; that is one of its standardly recognized ... features” (2004: 521).

I agree that global supervenience should allow intraworld variation, but is FPP the right way to express this variation? As it turns out, there are two ways to interpret the constraint of intraworld variation. According to one understanding – which is Leuenberger's – two objects can be distinguishable in their A-properties even if they are indistinguishable in all their B-features, even having the same maximal B-properties. FPP indeed allows this kind of variation, but, as scenario (IV) shows, SGS is inconsistent with this variation. Thus if this understanding is the correct one, SGS lacks an essential feature of a notion of global supervenience.
But there is another understanding of intraworld variation according to which two objects can be distinguishable in their A-properties even if they are indistinguishable in their "intrinsic" B-properties. This interpretation of intraworld variance is consistent with SGS: if SGS holds, then the two objects will be distinguishable in their "extrinsic" B-properties. Thus the friends of SGS can accept the idea of intraworld variation without accepting FPP. They can accept the idea that two objects can have the same "intrinsic" B-properties and different A-properties, but reject FPP, which allows the objects to be also indistinguishable in their maximal B-properties. The friends of SGS can still adopt FPP*, which is consistent with this idea of intraworld variation, instead of FPP, which is not.

The first interpretation of intraworld variation is motivated by those who claimed that intraworld variation is inconsistent with the notion of dependence (e.g., Kim 1990); based on these claims, Bennett (2004) and I (Shagrir 2002) suggested that people often thought of global supervenience in terms of IGS. These people might have thought that this understanding of intraworld variation makes global supervenience sufficiently distinct from the individual notion of strong supervenience. These points are well taken; yet the important issue, it seems to me, is not the imprecise ways that people characterized global supervenience. The important issue is which characterization does more justice to the notion of global supervenience. And it is the notion of SGS that both captures the idea that global supervenience reflects dependence and allows the intraworld variation of individuals. (The issue of whether SGS is sufficiently distinct from SIS is the topic of section 4.3).

The second interpretation is motivated by externalist ideas, e.g., in philosophy of mind, which, as we saw, constituted much of the philosophical motivation of global
supervenience. These externalists argue that two individuals can differ in their mental properties (A-properties) even though they have exactly the same "intrinsic" physical properties (intrinsic B-properties); in this sense they accept the principle of intraworld variance (under the second interpretation). Still, these externalists also think that the mental is dependent on the physical, namely, that the mental difference is accompanied by difference in physical properties and relations in the physical world.\footnote{See, e.g., Davidson (1987): "What I take Burge’s and Putnam’s imagined cases to show (and what I think the Swampman example shows more directly) is that people who are in all relevant physical respects similar (or ‘identical’ in the necktie sense) can differ in what they mean or think… But of course there is something different about them, even in the physical world; their causal histories are different".}

This idea, as we saw, can be made consistent only with the second interpretation of intraworld variation, which SGS expresses quite nicely.

The conclusion is that SGS does, after all, respect an intraworld variation of individuals, and does satisfy a finest partition principle, i.e., FPP*. In this respect, SGS is an adequate explication of the notion of global supervenience. It is true that SGS does not satisfy another finest partition principle, i.e., FPP, but this is a virtue, not a vice. For if you uphold FPP you lose dependence.

4.3 Bennett. Karen Bennett (2004) argues that "SGS has no metaphysically distinctive role to play" (2004:506). This is because "SGS and strong [SIS] are merely notational variants of one another. Every claim made with the one can also be made with the other" (2004:521). There are cases, such as in scenario (III), where we can express the dependence of A on B in terms of the SGS of A on B, but not in terms of the SIS of A on B. But as Bennett points out, we can express the dependence of A on B in terms of SIS; we just have to enlarge the B-base to include extrinsic properties. Indeed, Stalnaker's theorem, which states the equivalence of the SGS of A on B and
the SIS of A on maximal B-properties ensures that everything said with SGS can be said with SIS when the base is enlarged to maximal B-properties. Bennett proves, moreover, that if the A-properties are intrinsic, we do not even have to bother with that, for in this case the SGS and SIS of A on B are equivalent.

It should be noted first that SGS has certain aesthetic advantages: it expresses more directly certain dependencies that involve features in the individuals' environment, and it does not invoke the complex notion of maximal property (this point is acknowledged by Bennett). But I want to claim that it also has some metaphysical virtues. As I point out elsewhere (Shagrir 2009b) the equivalence theorems of SIS and SGS are formulated with respect to properties, intrinsic and extrinsic, but they do not extend to relations. In particular, when A and B include relations, it is not true that the SGS of A on B is equivalent to the SIS of maximal A-properties on maximal B-properties. Something is lost in the conversion of relations to maximal properties. Here is the gist of the argument:

Consider the following scenario (V):

(V) \( w_1 \)

\begin{align*}
Rab, Rba, Rcd, Rdc \\
Mab, Mbc, Med, Mda
\end{align*}

We can note that the strong global supervenience of \{M\} on \{R\} fails. The function \( f \) from \( w_1 \) onto \( w_1 \), which maps \( a \) to \( b \), \( b \) to \( a \), \( c \) to \( c \) and \( d \) to \( d \), is \{R\}-isomorphism. Yet \( f \) is not \{M\}-isomorphism as Mbc, but \( \neg M(f(b)f(c)) \).
What about SIS? The individual notions of supervenience are not defined over relations; we thus have to convert relations to properties. We can see, however, that the conversion to maximal properties will not do.\textsuperscript{22} The maximal R-property of $x$ is:

There are exactly three objects, $y$, $z$, and $v$, in the world other than $x$. $x$ stands in relation $R$ to $y$, $y$ stands in relation $R$ to $x$, $z$ stands in relation $R$ to $v$, $v$ stands in relation $R$ to $z$. None of the other pairs stand in $R$-relation.

The maximal M-property of $x$ is:

There are exactly three objects, $y$, $z$, and $v$, in the world other than $x$. $x$ stands in relation $M$ to $y$, $y$ stands in relation $M$ to $z$, $z$ stands in relation $M$ to $v$, $v$ stands in relation $M$ to $x$. None of the other pairs stand in $M$-relation.

Scenario (V) is thus consistent with the SIS of the maximal M-properties on the maximal R-properties: The objects $a$, $b$, $c$ and $d$ have exactly the same maximal R-property but also the same maximal M-property (We can stipulate that in all worlds that have exactly four objects, each having this maximal R-property, we get this pattern of M-relations. In none of the other worlds do objects stand to each other in M-relation).

**Corollary** (Theorem 12): The strong global supervenience of $A$ on $B$ (when $A$ and $B$ include relations) is not equivalent to the strong individual supervenience of the maximal $A$-properties on the maximal $B$-properties (where relations are expressed in the framework of maximal properties).

How does this result relate to the issue of dependence? We can notice that SGS, when defined over relations, captures an important fact about the non-dependence of the M-relations on the R-relations. That $\{M\}$ does not SGS on $\{R\}$ indicates that $\{M\}$ does not depend on $\{R\}$. The reason there is no dependence here is that we have two objects, $a$ and $b$, that are indistinguishable in their R-relations: they

\textsuperscript{22} Elsewhere (Shagrir 2009b), I consider other conversions to relations; these are far less effective than the conversion to maximal properties.
stand in relation R only to each other. Still, there is some difference among them with respect to their M-relations. The difference is that \(a\) stands in relation M to \(b\), but \(b\) stands in a relation M not to \(a\), but to \(c\). It thus follows that nothing in the R-relations of \(a\) and \(b\) determines that, or explains why, \(b\), and not \(a\), stands in M-relations to \(c\), and why Mab but not Mba.

SIS does not capture the non-dependence of \(\{M\}\) on \(\{R\}\), because the set of maximal M-properties strongly individually supervenes on the set of maximal R-properties. However, it would be a mistake to conclude that SIS is too weak, in the sense that it allows non-dependence scenarios such as (V). For SIS here is defined not over M-relations and R-relations, but over maximal M-properties and R-properties. With respect to these properties, SIS is exactly right: for the maximal M-properties does depend on the maximal R-properties; in particular, there is no difference between \(a\) and \(b\) with respect to their maximal M-properties. The trouble with SIS is that not being defined over relations, it sometimes cannot reflect (non)dependence of sets that include relation. If we want to express these (non)dependencies, we have to switch to SGS.

We can conclude that SGS has a metaphysically distinct role to play. We can say something about (in)dependence with SGS that we cannot say with SIS. After all, we can claim, using SGS, that \(\{M\}\) does not depend on \(\{R\}\); but we cannot say this with SIS.
5. Conclusion

“Concepts of supervenience” highlights three notions, those of strong, weak and global supervenience. Kim has argued that only the strong kind (SIS) can secure dependence; but others have kept exploring the potential and the varieties of global supervenience. The weak (WGS) and intermediate (IGS) versions of global supervenience turned out to be too weak to serve the dependence claims, and the strong version (SGS) has attracted many sorts of criticism. I have attempted to show here that the complaints about SGS are not justified. SGS does establish dependence, and is expressive enough to distinguish between relevant and irrelevant determining properties. It is an adequate explication of the “original” global supervenience (at least if we think of it as a notion of dependence). Lastly, SGS has major advantages over its individual counterpart (SIS). The definition of SGS is elegant and simple and does not invoke maximal properties. And SGS can serve to advance metaphysical claims about (in)dependence of relations, that cannot be advanced by SIS. I conclude that SGS is the most useful notion of supervenience we have.

Acknowledgment: Thanks to Vera Hoffmann, and to two anonymous referees of this Journal. This research was supported by the Israel Science Foundation, grant 725/08.

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